National Exams December 2013
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. 20 marks
3. 20 marks
4. (a) 10 marks, (b) 10 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. (a) Solve the initial value problem

$$y'' - y' - 6y = 2e^{-2t}, \qquad y(0) = 0, \quad y'(0) = 1/2.$$

   Note that $'$ denotes differentiation with respect to t.

   (b) Find the general solution, $y(x)$, of the differential equation

$$xy'' - y' = 3x^2e^x.$$

   Note that $'$ now denotes differentiation with respect to x.

2. Find out what type of conic section (e.g., parabola, hyperbola, or ellipse) the following quadratic form represents and transform it to principal axes. (That is, find new variables u and v so that $Q = au^2 + bv^2$.)

$$Q = -2x^2 + 12xy + 7y^2 = 156.$$

3. Find the minimum value of the function $F(x, y, z) = x - y + 2z$ subject to the constraint $x^2 + 3y^2 + 2z^2 = 5$.

4. Let P be the plane passing through the three points (2,1,-2), (1,2,0) and (1,0,-1).

   (a) Find an equation representing the plane P.

   (b) Find the line of intersection between the plane P and the plane $x + y - 2z = 3$

5. Let S be the surface of the region defined by $x^2 + 4y^2 \leq 1$, $x \geq 0$, $y \geq 0$, $0 \leq z \leq 4$, and let F be the vector function $F(x, y, z) = (y^3, x^3, z^3)$. Evaluate the integral of F over the surface S.

6. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$ where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z - 2x + y = 1$, travelled clockwise as viewed from the positive z-axis, and $\mathbf{v}$ is the vector function $\mathbf{v} = z + x\mathbf{i} - 2y\mathbf{j} + y^2\mathbf{k}$.

7. At what angle does the line represented parametrically by $x = 2 - t$, $y = t$, $z = 2 + 2t$ intersect the hyperboloid $z = 8 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine.

8. Find the equation of motion of the mass-spring system corresponding to the following equation and initial conditions:

$$y'' + 2y' + 2y = \cos(t), \qquad y(0) = 1.2, \; y'(0) = 1.4.$$

Note that $'$ denotes differentiation with respect to x.