National Exams December 2015
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. (a) 10 marks, (b) 10 marks
3. (a) 10 marks, (b) 5 marks, (c) 5 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solutions of the following differential equations:

   (a) $y' + xy = 2xe^{-x^2}$,

   (b) $y'' + y' - 6y = 0$.

   Note that in each case, $'$ denotes differentiation with respect to x.

2. Solve the following initial value problems:

   (a) $y' + 2ty^2 = 0, \quad y(1) = 2,$

   (b) $y'' - 12y' + 45y = 18\cos(3t), \quad y(0) = 0, \quad y'(0) = 0.$

   Note that in each case, $'$ denotes differentiation with respect to t.

3. Consider the two lines defined as follows:

   $x = 3 + 2t, \quad y = 3, \quad z = 1 - t$, (parameter t);
   $x = s, \quad y = 1 - 2s, \quad z = 2 + s$, (parameter s).

   (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.

   (b) Find a third line orthogonal to both lines.

   (c) Is there a plane containing both lines? If so, find an equation for that plane.

4. Find the maximum and minimum values of $f(x, y, z) = x + y - 2z$ over the ellipse $x^2 + y^2 + 4z^2 = 1$.

5. Let S be the boundary of the region enclosed by the paraboloid $z = 4 - x^2 - y^2$ and the plane $z = 0$ and let

   $$\mathbf{F}(x, y, z) = x^2\mathbf{i} - 2xy\mathbf{j} + x^2z\mathbf{k}.$$

   Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$, where $\mathbf{n}$ is the unit outward normal on S.

6. At what angle does the line represented parametrically by $x = 1 - t, \quad y = t, \quad z = 2 + 3t$ intersect the hyperboloid $z = 4 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine.

7. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$ where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z + 2x - y = 3$, travelled counterclockwise as viewed from the positive z-axis, and $\mathbf{v}$ is the vector function $\mathbf{v} = x\mathbf{i} + (x - y)\mathbf{j} + yz\mathbf{k}$.

8. Find the volume of the solid region inside the ellipsoid

   $$x^2 + y^2 + 4z^2 = 5$$

   and above the cone

   $$z = \sqrt{x^2 + y^2}.$$