National Exams December 2019
04-BS-1, Mathematics
3 hours Duration

<u>Notes:</u>

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.

2. A Casio or Sharp approved calculator is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. 20 marks

2. 20 marks

3. 20 marks

4. 20 marks

5. 20 marks

6. 20 marks

7. 20 marks

8. 20 marks

1. Solve the following initial value problem

$$y'' - 12y' + 45y = 18\cos(3t), \quad y(0) = 0, \quad y'(0) = 0.$$

Note that, $'$ denotes differentiation with respect to t.

2. Find the general solution, $y(x)$, of the differential equation

$$y' + 2xy = e^{-x^2}\sec(2x)$$

Note that $'$ denotes differentiation with respect to x.

3. Find the minimum value of the function $F(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $x + y - z + 1 = 0$

4. Find the general solution to the following system of differential equations.

$$\frac{dx}{dt} = 4x + 2y,$$

$$\frac{dy}{dt} = 3x - y + e^{-2t}.$$

5. Let $f(x, y) = 1 + x\ln(xy - 5)$. Find a formula for the plane tangent to the surface $z = f(x, y)$ at the point $(2, 3)$ and use the tangent plane to approximate $f(2.1, 2.95)$.

6. Find the volume of the solid region inside the ellipsoid

$$x^2 + y^2 + 4z^2 = 5$$

and above the cone

$$z = \sqrt{x^2 + y^2}.$$

7. Find the work done by the field $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y\mathbf{j} - z\mathbf{k}$ in moving a particle from the point $(0, 2, 0)$ to the point $(3\pi, 0, 2)$ along the path $x = 6t$, $y = 2\cos t$, $z = 2\sin t$.

8. Find the surface area of that portion of the surface $z = 1 - \sqrt{x^2 + y^2}$ that lies in the first octant.