National Exams

December 2018

04-BS-1, Mathematics

3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. An APPROVED Casio or Sharp CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. 20 marks
3. 20 marks
4. (a) 7 marks (b) 7 marks (c) 6 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solutions of the following differential equations:

   (a) $y' + xy = 2xe^{-x^2}$,

   (b) $y'' + y' - 6y = 0$.

   Note that in each case, $'$ denotes differentiation with respect to x.

2. Find the general solution, $x(t)$, of the differential equation $x'' + 4x = 3\cos 2t + 4\cos 3t$. Note that $'$ denotes differentiation with respect to t.

3. Find the maximum and minimum values of $f(x, y, z) = 3x + 2y^2 + z$ over the ellipsoid $3x^2 + y^2 + z^2 = 1$.

4. Consider the two lines defined as follows:
   $x = 3 + 2t, \quad y = 3, \quad z = 1 - t,$ (parameter t);
   $x = s, \quad y = 1 - 2s, \quad z = 2 + s,$ (parameter s).

   (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.

   (b) Find a third line orthogonal to both lines.

   (c) Is there a plane containing both lines? If so, find an equation for that plane.

5. At what angle does the line represented parametrically by $x = 1 - t, \; y = t, \; z = 2 + 3t$ intersect the hyperboloid $z = 4 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine.

6. Evaluate the surface integral $\iint_S \mathbf{F} \cdot dS$ where $\mathbf{F}(x, y, z) = xz\mathbf{i} - 2y\mathbf{j} + 3x\mathbf{k}$ and S is the surface of the region bounded above by the paraboloid $z = 4 - x^2 - y^2$ and below by the plane $z = 0$.

7. Find the work done by the field $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y\mathbf{j} - z\mathbf{k}$ in moving a particle from the point $(0, 2, 0)$ to the point $(3\pi, 0, 2)$ along the path $x = 6t, \; y = 2\cos t, \; z = 2\sin t$.

8. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 1 + y$, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2x\mathbf{j} + 2x\mathbf{k}$. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$. Assume a clockwise orientation for the curve when viewed from above.