National Exams May 2013
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.

2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET *written on both sides*.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. 20 marks

2. (a) 7 marks, (b) 7 marks, (c) 6 marks

3. (a) 6 marks, (b) 14 marks

4. 20 marks

5. 20 marks

6. (a) 7 marks, (b) 7 marks, (c) 6 marks

7. 20 marks

8. 20 marks

1. Solve the initial value problem

$$y'' - 4y' + 8y = 5\cos(2t), \qquad y(0) = 0,\ y'(0) = 0,$$

   where $'$ denotes differentiation with respect to t.

2. Find the general solution of the following differential equations:

   (a) $y' + 2xy = 2xe^{-x^2}$,

   (b) $y' + 2xy^2 = 0$,

   (c) $y'' - 2y' + 3y = 0$.

3. (a) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{pmatrix} 4 & 3 \\ -1 & 0 \end{pmatrix}$.

   (b) Solve the system of differential equations

   $$\frac{dx}{dt} = 4x + 3y,$$
   $$\frac{dy}{dt} = -x + t,$$

   subject to the initial conditions $x(0) = 2$, $y(0) = -1$.

4. Let $f(x, y) = 1 + x \ln(xy - 5)$. Find a formula for the plane tangent to the surface $z = f(x, y)$ at the point $(2, 3)$ and use the tangent plane to approximate $f(2.1, 2.95)$.

5. Find the surface area of that portion of the surface $z = 1 - \sqrt{x^2 + y^2}$ that lies in the first octant.

6. Consider the two lines defined as follows:

   $$x = 3 - 2t, \quad y = 3, \qquad z = 3 - t, \text{ (parameter } t\text{)};$$
   $$x = s, \qquad\quad y = 1 - 2s, \quad z = -s, \text{ (parameter } s\text{)}.$$

   (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.

   (b) Find a third line orthogonal to both lines.

   (c) Is there a plane containing both lines? If so, find an equation for that plane.

7. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$ where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z + 2x - y = 3$, travelled counterclockwise as viewed from the positive z-axis, and $\mathbf{v}$ is the vector function $\mathbf{v} = x\mathbf{i} + (x - y)\mathbf{j} + yz\mathbf{k}$.

8. Find the volume of the solid region inside the ellipsoid

$$x^2 + y^2 + 4z^2 = 5$$

   and above the cone

$$z = \sqrt{x^2 + y^2}.$$