

National Exams December 2016

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

Notes:

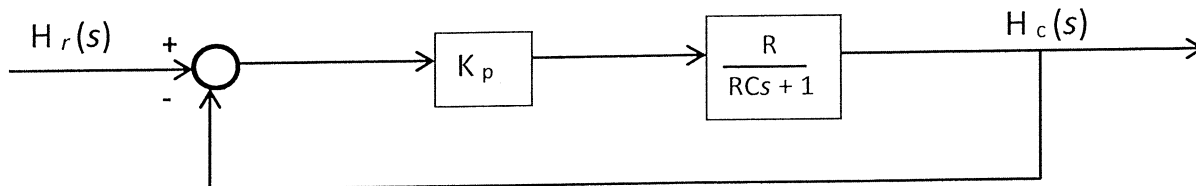
1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio or Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.
3. Any four (4) questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1) Find the impulse response for systems with the following transfer function.

$$a) G(s) = \frac{100}{(s+2)(s+5)(s+5)}$$

$$b) G(s) = \frac{1}{.04s^2 + .08s + 1}$$

2) A block diagram for a liquid-level system is shown in the diagram below. Determine the steady-state response $h_{CSS}(t)$ if a step reference input $H_R u(t)$ is applied.



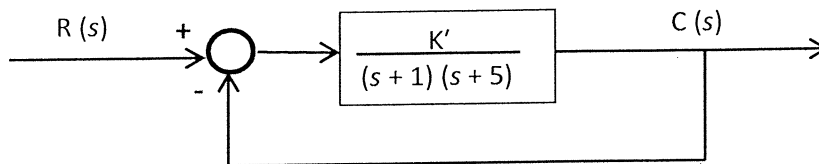
3) Plot Bode diagrams for the following open-loop transfer functions. For the attenuation curve in each case plot the asymptotic approximation first and then put in the actual curve by making corrections at the break points or calculating the actual value of the attenuation at specific frequencies.

$$a) G(s)H(s) = \frac{15(0.001s + 1)}{0.01s + 1}$$

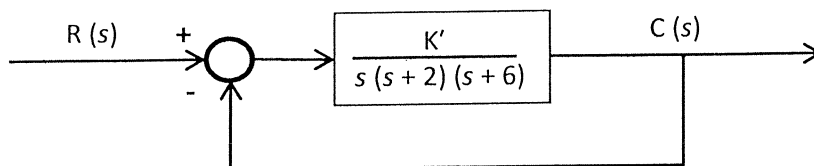
$$b) G(s)H(s) = \frac{25}{(0.1s + 1)(0.2s + 1)(0.01s + 1)}$$

- 4) For each of the systems shown in the accompanying illustration, sketch the root-locus plot. Determine the value of K' required for a damping ratio of 0.5. With K' set, what will be the maximum time constant and the damped natural frequency?

a)



b)



- 5) a) Using the Routh-Hurwitz criterion, investigate the stability of the following characteristic equation:

$$S^5 + S^4 + 2S^3 + S^2 + S + K = 0$$

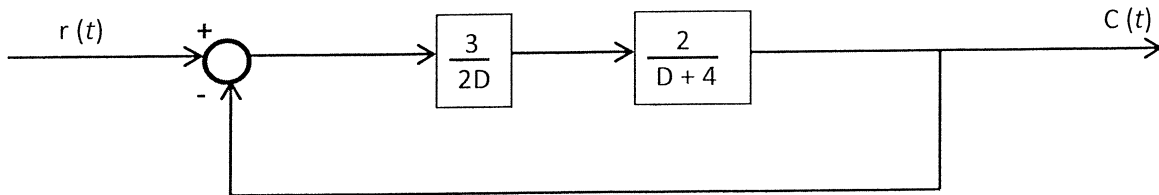
- b) What is the unit step response of a system whose transfer function is given by

$$P(s) = \frac{(S + 2)}{(S + 0.5)(S + 4)}$$

- c) A system is designed to give satisfactory performance when a particular amplifier gain K has the value 2. Determine how much this gain can vary before the system becomes unstable if the characteristic equation is:

$$S^3 + (4 + K) S^2 + 6 S + 16 + 8 K = 0$$

- 6) Determine the position, velocity and acceleration error constants, and the steady-state error to a unit step, a unit ramp, and a unit parabolic input for the system shown in the figure below.



where $D = \frac{d}{dt}$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$	$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$ (1)	$\frac{ae^{at} - be^{bt}}{a - b}$	$\frac{s}{(s - a)(s - b)}$ (19)
$e^{at} f(t)$	$F(s - a)$ (2)	$t e^{at}$	$\frac{1}{(s - a)^2}$ (20)
$\mathcal{U}(t - a)$	$\frac{e^{-as}}{s}$ (3)	$t^n e^{at}$	$\frac{n!}{(s - a)^{n+1}}$ (21)
$f(t - a)\mathcal{U}(t - a)$	$e^{-as}F(s)$ (4)	$e^{at} \sin kt$	$\frac{k}{(s - a)^2 + k^2}$ (22)
$\delta(t)$	1 (5)	$e^{at} \cos kt$	$\frac{s - a}{(s - a)^2 + k^2}$ (23)
$\delta(t - t_0)$	e^{-st_0} (6)	$e^{at} \sinh kt$	$\frac{k}{(s - a)^2 - k^2}$ (24)
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$ (7)	$e^{at} \cosh kt$	$\frac{s - a}{(s - a)^2 - k^2}$ (25)
$f'(t)$	$sF(s) - f(0)$ (8)	$t \sin kt$	$\frac{2ks}{(s^2 + k^2)^2}$ (26)
$f^{(n)}(t)$	$s^n F(s) - s^{(n-1)}f(0) - \dots - f^{(n-1)}(0)$ (9)	$t \cos kt$	$\frac{s^2 - k^2}{(s^2 + k^2)^2}$ (27)
$\int_0^t f(x)g(t - x)dx$	$F(s)G(s)$ (10)	$t \sinh kt$	$\frac{2ks}{(s^2 - k^2)^2}$ (28)
t^n ($n = 0, 1, 2, \dots$)	$\frac{n!}{s^{n+1}}$ (11)	$t \cosh kt$	$\frac{s^2 - k^2}{(s^2 - k^2)^2}$ (29)
t^x ($x \geq -1 \in \mathbb{R}$)	$\frac{\Gamma(x + 1)}{s^{x+1}}$ (12)	$\frac{\sin at}{t}$	$\operatorname{arctan} \frac{a}{s}$ (30)
$\sin kt$	$\frac{k}{s^2 + k^2}$ (13)	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$ (31)
$\cos kt$	$\frac{s}{s^2 + k^2}$ (14)	$\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$ (32)
e^{at}	$\frac{1}{s - a}$ (15)	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$ (33)
$\sinh kt$	$\frac{k}{s^2 - k^2}$ (16)		
$\cosh kt$	$\frac{s}{s^2 - k^2}$ (17)		
$\frac{e^{at} - e^{bt}}{a - b}$	$\frac{1}{(s - a)(s - b)}$ (18)		