National Exams May 2012
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. (a) 6 marks, (b) 6 marks, (c) 8 marks
3. 20 marks
4. 20 marks
5. (a) 16 marks, (b) 4 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. (a) Solve the initial value problem

$$y'' + 9y = 6\cos(3t), \qquad y(0) = 5,\ y'(0) = 0.$$

(b) Find the general solution of the differential equation

$$y' + 2xy = 2xe^{-x^2}$$

2. Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \\ -10 & -4 & -2 \end{pmatrix}$$

(a) Show that $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ is an eigenvector of A and find the associated eigenvalue.

(b) Show that 3 is an eigenvalue of A and find an associated eigenvector.

(c) Solve the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the function $\mathbf{x}(t)$.

3. Find the centre of mass of the solid bounded by the two paraboloids $z = 2x^2 + 2y^2$ and $z = 3 - x^2 - y^2$ whose density is $\rho(x, y, z) = 2z$.

4. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$, where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $z = 1 + y - 2x$, travelled clockwise as viewed from the positive z-axis, and $\mathbf{v}$ is the vector function $\mathbf{v} = 2z^2\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$.

5. Consider the quadratic form $2x^2 - 6xy + 2y^2 = 13$.

(a) Transform the quadratic form to principal axes.

(b) What type of conic section is represented by the above quadratic form?

6. Use Lagrange multipliers to find the volume of the largest box with faces parallel to the coordinate planes that can be inscribed in the ellipsoid

$$16x^2 + 4y^2 + 9z^2 = 144.$$

7. Find the line tangent to the intersection of the surfaces

$$x^2 + y^2 + z^2 - 3y + 2z = 0$$

and

$$x^2 - y^2 - 2z = 5$$

at the point $(\sqrt{2}, 1, -2)$.

8. Find the surface area of that portion of the surface $z = 1 - \sqrt{x^2 + y^2}$ that lies in the first octant.