# PROFESSIONAL ENGINEERS ONTARIO 

National Examinations - December 2016<br>07-Mec-A5, Electrical \& Electronics Engineering<br>Mechanical Engineering

## 3 hours duration

Name [print]:
Signature:

Notes:
[1] If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.
[2] Candidates may use one of two calculators, the Casio or Sharp approved models. This is a closed book examination.
[3] This examination consists of the front page and 8 numbered pages.
[4] Any five (5) questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
[5] Each question is of equal value.
[6] Clarity and organization of answers are important.
[7] The candidate is required to sign this examination paper and submit it with the solution booklets.
[8] $\pi=3.14159$
$1 \mathrm{hp}=746 \mathrm{~W}$

$$
\mu_{0}=4 \pi \times 10^{-7} \mathrm{Hm}^{-1}
$$

## QUESTION 1

Consider the amplifier circuit shown in Figure 1. Assume an average DC current gain $\beta=100$ for the npn transistor.
[a] Determine the values of $R_{E}$ and $R_{C}$ required for an operating point of $I_{C}=2 \mathrm{~mA}$ and $V_{C E}=6 \mathrm{~V}$.
[b] Sketch the $I_{C}$ vs $V_{C E}$ characteristic and draw the dc load line.
[c] For $R_{L}=3 k \Omega$, draw the ac load line and estimate the output voltage $v_{o}$ for an input current $\mathrm{i}_{\mathrm{b}}=10 \sin \omega t \mu \mathrm{~A}$.


Figure 1 Transistor Circuit

| Component List |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
| $\mathrm{R}_{1}=10 \mathrm{k} \Omega$ | $\mathrm{R}_{2}=30 \mathrm{k} \Omega$ | $\mathrm{V}_{\mathrm{CC}}=15 \mathrm{~V}$ |  |  |

## QUESTION 2

This question consists of two parts which are not necessarily related.

## Part I: Design

Develop the truth table for a 2-input exclusive or gate and write the Boolean algebra expression for the output Y as a function of the inputs $\mathrm{A}, \mathrm{B}$.
You are provided with quantity six 2 -input nor gates. Design the gate array to implement the 2 -input exclusive or function.

## Part II: Analysis

A combinational logic circuit is shown in Figure 2.
[a] Write a general Boolean algebra expression for the output C as a function of the inputs $\mathrm{A}, \mathrm{B}, \mathrm{K}_{0}$, and $\mathrm{K}_{1}$.
[b] Apply DeMorgan's theorems and simplify the expression obtained in [a].
[c] For each of the 4 possible combinations of $K_{0}, K_{1}$, reduce the expression for $C$ to its simplest form.


Figure 2 Circuit Schematic

## QUESTION 3

A linear dc machine consisting of a conducting bar resting on two conducting rails is shown in Figure 3. The magnetic flux density is 1 T , directed into the page, the resistor R is $0.05 \Omega$, the rail separation is 1 metre, and the battery voltage $\mathrm{V}_{0}$ is 2 V . Assume negligible friction between the bar and rails.
[a] Assume that the bar is stationary at time $t=0$ when the switch $S_{1}$ is closed. What is the magnitude and direction of the force on the bar at the start? What is the initial current flow? What is speed of the bar assuming that no mechanical load is applied to the bar?
[b] If the bar is loaded down with a force of 20 N opposite to the direction of motion, determine the new steady state speed. Calculate the power delivered by the battery, the power delivered to the mechanical load and the power lost to heat in the resistance R . What is the efficiency of the machine under these circumstances?
[c] The loading force in part [b] is removed and a mechanical pulling force of 10 N directed to the right is applied to the bar. In steady state, determine the speed, the power taken from the mechanical source, the power delivered to the battery and the power lost to heat in the resistance $R$. What is the efficiency of the machine under these circumstances?


Figure 3 Top View of Linear dc Machine

## QUESTION 4

Consider the magnetic circuit of a transformer shown in Figure 4. Infinite relative permeability can be assumed for the iron core.


Figure 4 Transformer
The following specifications apply.

| $\mathrm{L}_{1}$ | $3.77 \times 10^{-2} \mathrm{~m}$ | $\mathrm{~A}_{1}$ | $0.02 \mathrm{~m}^{2}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{~L}_{2}$ | $7.54 \times 10^{-2} \mathrm{~m}$ | $\mathrm{~A}_{2}$ | $0.02 \mathrm{~m}^{2}$ |
| $\mathrm{N}_{1}$ <br> [primary] | 200 turns | $\mathrm{N}_{2}$ <br> [secondary] | 20 turns |

When a dc voltage equal to 10 mV is applied to the primary, the measured primary current is 100 mA . When a dc voltage of 0.1 mV is applied to the secondary winding, the measured secondary current is 100 mA .
Assume that leakage inductances and eddy current and hysteresis losses are negligible; consider an operating frequency of 1000 Hz .
[a] Draw the equivalent circuit of the transformer referred to the primary and calculate component values.
[b] A transducer with an impedance of $0.078 \Omega$ is connected across the secondary of the transformer, an amplifier is connected to the primary. Calculate the output impedance of the amplifier to give maximum power transfer to the load.

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## QUESTION 5

Consider the circuit shown in Figure 5 which has been designed using ideal operational amplifiers ( $U_{1}$ to $U_{3}$ ) with infinite bandwidth and infinite open loop gain. In the schematic, $a, b$ and $c$ are constants. You will note that $U_{3}$ is configured as a basic difference amplifier which has a transfer function given by:

$$
E_{0}=c\left(e_{y}-e_{x}\right)
$$

where $e_{y}$ and $e_{x}$ are the potentials at points $y$ and $x$ respectively.
In the derivation of the transfer function for such circuits, one can assume:
[i] Zero differential voltage between the input terminals of the operational amplifier,
[ii] Zero current flows into either input terminal of the operational amplifier.

Applying the principle of superposition, derive an expression for the transfer function of the total circuit [ $E_{O}$ as a function of $E_{1}, E_{2}$ ].

Hint: Let $E_{2}=0$, and solve for the potentials at points $x$ and $y$ for input $E_{1}$. Let $E_{1}=0$, and again solve for the potentials at points $x$ and $y$ for input $E_{2}$. Calculate the resultant output $E_{0}$ for both $E_{1}$ and $E_{2}$ inputs.


Figure 5 Circuit Schematic

## QUESTION 6

This question consists of two parts which are not necessarily related.

## Part I

A 3 phase, $300 \mathrm{hp}, 12$ pole wound rotor induction motor is operated from a 60 Hz source. The per phase rotor resistance $r_{2}$ was measured and found to be $0.04 \Omega$. At full load, the speed of the motor is 582 rpm .

At full load, determine:
[a] The speed of the magnetic field in revolutions per minute.
[b] The slip of the rotor.
[c] The frequency of the rotor currents.
[d] The angular velocity of the stator field with respect to the stator.
[e] The angular velocity of the stator field with respect to the rotor.
[f] The angular velocity of the rotor field with respect to the rotor.
[g] The angular velocity of the rotor field with respect to the stator.

## Part II

In the normal operating region of an induction motor, torque is a linear function of slip. A test was performed on a 3 phase, 8 pole squirrel cage induction motor which is operated from a 60 Hz source and it was found that it developed a torque of $3 \mathrm{~N} . \mathrm{m}$ at a speed of 810 rpm .
The induction motor is used to drive a load which requires a torque which is a linear function of speed. In another test, it was found that the torque required by the load was $0.5 \mathrm{~N} . \mathrm{m}$ at a speed of 435 rpm .
[a] Sketch the speed-torque characteristics for the motor and load.
[b] Calculate the operating point for the motor-load system.

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\text { Page }-6 \text { of } 8
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## QUESTION 7

Consider the RC circuit shown in Figure 7. The switch $S_{1}$ is closed at time $t=0$, connecting the dc supply, $\mathrm{V}_{\mathrm{I}}$, to the network.
[a] Derive an expression for the voltage drop across the capacitor $\mathrm{C}_{1}$ as a function of time.
[b] Derive an expression for the current delivered to the capacitor $C_{1}$ as a function of time.


Figure 7 RC Circuit

| COMPONENT LIST |  |  |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{R}_{1}=30 \mathrm{k} \Omega$ | $\mathrm{R}_{2}=30 \mathrm{k} \Omega$ | $\mathrm{V}_{\mathrm{I}}=10 \mathrm{~V}$ | $\mathrm{C}_{1}=3 \mu \mathrm{~F}$ |

## QUESTION 8

## Part I

Consider the circuit shown in Figure 8a. A one volt (rms) ac voltage source, given by $\mathrm{V}=1 \angle 0^{\circ}$ is connected to a parallel R-L network. The resistor R has a value of $1 \Omega$; the impedance of the inductor is $\mathrm{j} 1 \Omega$.
Calculate: [a] the current through $\mathrm{R}, \mathrm{I}_{1} ;[\mathrm{b}]$ the current through $\mathrm{L}, \mathrm{I}_{2} ;[\mathrm{c}]$ the total current into the network, $\mathrm{I}_{\mathrm{T}} ;[\mathrm{d}]$ the real power delivered to the load; $[\mathrm{e}]$ the system power factor. Note: The currents for parts [a] - [c] should be expressed as phasor quantities, of the form: $\mathrm{I} \angle \varphi^{\circ}$.


Figure 8a Circuit for Part I

## Part II

A capacitor is added in parallel to the inductor as shown in Figure 8 b . The impedance of the capacitor is $-\mathrm{j} 1 \Omega$.
Calculate: [a] the current through $\mathrm{R}, \mathrm{I}_{1} ;[\mathrm{b}]$ the current through $\mathrm{L}, \mathrm{I}_{2} ;[\mathrm{c}]$ the current through $\mathrm{C}, \mathrm{I}_{3}$; [d] the total current into the network, $\mathrm{I}_{\mathrm{T}}$; [e] the real power delivered to the load; [f] the system power factor. Note: The currents for parts [a] - [d] should be expressed as phasor quantities, of the form: $\mathrm{I} \angle \varphi^{\circ}$.


Figure 8b Circuit for Part II

