National Exams May 2015

## 04-Chem-A6, Process Dynamics \& Control

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK EXAM.

Any non-communicating calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper.

The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value.
5. Most questions require an answer in essay format: Clarity and organization of the answer are important.

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## PROBLEM 1 (20\%)

The Calrod heating element shown in the drawing transfers heat largely by a radiation mechanism. If the rate of electrical energy input to the heater is $Q$ and the rod temperature and ambient temperatures are, respectively, $T$ and $T_{a}$, then an appropriate unsteady-state model for the system is

$$
\mathrm{mC} \frac{\mathrm{dT}}{\mathrm{dt}}=\mathrm{Q}-\mathrm{k}\left(\mathrm{~T}^{4}-\mathrm{T}_{\mathrm{a}}^{4}\right)
$$

$m$ is the mass of the heater, $C$ is specific heat and $k$ is radiation coefficient.
$(15 \%)$ a) Linearize and then find the transfer functions relating $\delta \mathrm{T}$ to $\delta \mathrm{Q}$ and $\delta \mathrm{T}$ to $\delta \mathrm{T}_{\mathrm{a}}$. (Be sure they are both in standard form, i.e. show gain and time constant.)

$(5 \%)$ b) If you were to design a proportional controller to control T by manipulating Q , what will be the sign of the controller to guarantee stability? Justify your answer.

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Problem \#2 (20\% total)

For the equation

$$
\frac{d^{2} y}{d t^{2}}+k \frac{d y}{d t}+2 y=x
$$

(10\%) (a) Find the transfer function and put it in the standard gain time constant form.
(10\%) (b) Discuss the response for values of $-20<\mathrm{k}<20$.
Specify for which values the response converges and where it will not. Write the form of the response without evaluating any coefficients.

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## PROBLEM 3 (20\%)

Consider the following system of equations:

$$
\begin{gathered}
\frac{d x_{1}}{d t}=-2.4048 x_{1}+7 u \\
\frac{d x_{2}}{d t}=0.8333 x_{1}-2.2381 x_{2}-1.117 u \\
y=x_{2}
\end{gathered}
$$

$(10 \%)$ a) Find the transfer function $Y(s) / U(s)$
$(10 \%)$ b) Solve for $y$ in response to a unit step change in $u$.

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PROBLEM 4 ( $20 \%$ )
A thermocouple is immersed in a well stirred bath of liquid. The geometry and properties of the thermocouple's material are as follows:

$$
\begin{aligned}
& \text { mass }=0.25 \mathrm{~g} \\
& \text { heat capacity }=1 \mathrm{cal} / \mathrm{g}{ }^{\circ} \mathrm{C}
\end{aligned}
$$

Heat transfer coefficient between the thermocouple and the liquid $=60 \mathrm{cal} / \mathrm{cm}^{2} \mathrm{~h}{ }^{\circ} \mathrm{C}$ surface area of the thermocouple $=1 \mathrm{~cm}^{2}$.
$(10 \%)$ a) Find the transfer function that relates the temperature of the thermocouple to the temperature in the liquid. Assume that there are no gradients in the thermocouple bead, no conduction through the thermocouple wires and the conversion from Millivolt to degrees occurs by a very fast reading device.
$(10 \%)$ b) If the temperature in the liquid changes according to the following diagram, calculate the temperature registered by the thermocouple as a function of time.


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## PROBLEM 5 (20\%)

A process described by the following transfer function:

$$
G(s)=\frac{5 e^{-2 s}}{10 s+1}
$$

Is to be controlled by an IMC (Internal Model Controller) controller. Time is in seconds.
$(10 \%)$ a) Show the block diagram of the closed loop. Calculate the IMC controller Gc ${ }^{*}$ and the classical feedback controller equivalent Gc (without assuming Pade approximation at this point).Assume that the IMC filter parameter is $\tau_{\mathrm{C}}=20 \mathrm{sec}$. Is the resulting Gc of PID form?
$(10 \%)$ b) Calculate the closed loop response for the controlled variable $\delta \mathrm{C}(\mathrm{t})$ for a unit step change in set point for the controller in item a) where Pade was not assumed and the model is assumed to be perfect.

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## PROBLEM 6 (20\%)



A pipe of length $L=10 \mathrm{~m}$ is feeding liquid into a tank. The speed of the liquid in the pipe is $u=1$ $\mathrm{m} / \mathrm{s}$. The level of liquid in the tank is h . The cross section area of the tank is $A=1 \mathrm{~m}^{2}$.
$(5 \%)$ a) Calculate the time delay in the pipe between inlet to outlet for the given velocity.
Assume that this delay remains constant for the rest of this problem despite changes in flowrate.
$(5 \%)$ b) Model the system, i.e. formulate differential equations to calculate $h(t)$ with respect to inlet velocity $u$ and find the open loop transfer function between $h$ to $u$.
$(10 \%) \mathrm{c})$ If the height h is controlled by manipulating the velocity u with a proportional controller with gain Kc , find the closed loop transfer function between the set point changes in $h$ to changes in $h$.

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Problem \#7 (20\% total)
A process given by:

$$
G_{p}=\frac{1}{s^{2}-s-2}
$$

is controlled by a proportional controller with gain $\mathrm{k}_{\mathrm{c}}$.
(a) Show a qualitative Nyquist plot (show only 2-3 key points and the general shape of the plot for this problem) for $\mathrm{k}_{\mathrm{c}}=1$. Assess the closed loop stability based on the Nyquist criterion.
(10\%) (b) Based on the Nyquist criterion, compute a range of $\mathrm{k}_{\mathrm{c}}$ values to obtain closed loop stability.

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## PROBLEM 8 (20\%)

A first order process is given by

$$
\mathrm{Gp}=\frac{1}{(\mathrm{~s}+5)}
$$

This process is controlled by a PI controller given by:

$$
\mathrm{Gp}=\mathrm{K}_{\mathrm{c}}\left(1+\frac{1}{\mathrm{~s}}\right)
$$

$(10 \%)$ a) Compute ranges of $\mathrm{K}_{\mathrm{c}}$ values for which the closed loop is stable.
$(10 \%)$ b) For a controller with gain $K_{c}=1$ compute the closed loop time response for a unit step change in the set point.

