

National Exams December 2011  
04-BS-1, Mathematics  
3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
  2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
  3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
  4. All questions are of equal value.
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Marking Scheme.

1. (a) 12 marks, (b) 8 marks
2. (a) 10 marks, (b) 10 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. (a) 3 marks, (b) 3 marks, (c) 14 marks
8. 20 marks

1. Let  $P$  be the plane passing through the three points  $(0,1,2)$ ,  $(1,3,-1)$  and  $(2,0,1)$ .

- (a) Find an equation representing the plane  $P$ .  
(b) Find the line of intersection between the plane  $P$  and the plane

$$x - 2y + z = 3$$

2. (a) Find the eigenvalues and the eigenvectors of the matrix

$$\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$$

- (b) Solve the system of differential equations

$$\begin{aligned} \frac{dx}{dt} &= 3x - 2y, \\ \frac{dy}{dt} &= x + y + e^{-2t}. \end{aligned}$$

subject to the initial conditions  $x(0) = 2$ ,  $y(0) = -1$ .

3. Find the solution,  $y(x)$ , of the differential equation

$$y'' + 9y = \sec 3x,$$

$y'(0) = 0$ ,  $y(0) = 1$ . Note that  $'$  denotes differentiation with respect to  $x$ .

4. Find an equation for the line tangent to the intersection of the surfaces

$$x^2 + y^2 - 6z = 11$$

and

$$4x^2 + y^2 + z^2 - 4y - 4z + 3 = 0$$

at the point  $(1, 0, 2)$ .

5. At what angle does the line represented parametrically by  $x = 2 - t$ ,  $y = t$ ,  $z = 2 + 2t$  intersect the hyperboloid  $z = 4 - x^2 + y^2$ ? You may leave your answer as an inverse sine or cosine.
6. Let  $S$  be the surface of the region defined by  $x^2 + 4y^2 \leq 1$ ,  $x \geq 0$ ,  $y \geq 0$ ,  $0 \leq z \leq 4$ , and let  $F$  be the vector function  $F(x, y, z) = (y^3, x^3, z^3)$ . Evaluate the integral of  $F$  over the surface  $S$ .
7. Let  $C$  be the curve formed by the intersection of the cylinder  $x^2 + y^2 = 9$  and the plane  $z = 1 + y - 2x$ , travelled clockwise as viewed from the positive  $z$ -axis, and let  $\mathbf{v}$  be the vector function  $\mathbf{v} = 4z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$ .
- (a) Evaluate the divergence of  $\mathbf{v}$
  - (b) Evaluate the curl of  $\mathbf{v}$
  - (c) Evaluate the line integral  $\oint_C \mathbf{v} \cdot d\mathbf{r}$ .

8. Find the general solution of the differential equation

$$x^2 y'' - 4xy' + 6y = 3x^4.$$

Note that  $'$  denotes differentiation with respect to  $x$ .